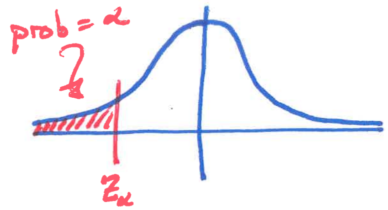
A Brief Note on Critical Values & Normal Dist.

In R the inverse of pnorm is qnorm

$$\text{pnorm}(z_\alpha) = \alpha \quad \Longleftrightarrow \quad z_\alpha = \text{qnorm}(\alpha) \qquad \Big\} \; P(Z < z_\alpha) = \alpha$$



Note: This is the negative of the textbook's z_α

Textbook: "$P(Z > z_\alpha) = \alpha$"

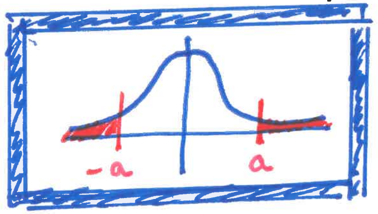
["p changes to q when it moves across = sign"]

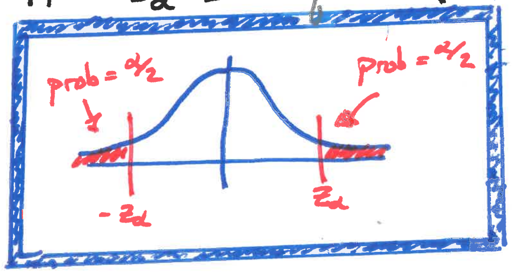
For extreme value computations, this is

$$P(|Z| > a) = 2\,\text{pnorm}(-a) \quad \Longleftrightarrow \quad P(|Z| > z_\alpha) = \alpha$$

$2 \cdot \text{pnorm}(-z_\alpha) = \alpha$

if $z_\alpha = -\text{qnorm}(\alpha/2)$





We will use these A LOT in the final sections of the course...

$$P(|Z| < a) = 1 - 2\,\text{pnorm}(-a) \quad \Longleftrightarrow \quad P(|Z| < z_\alpha) = \alpha$$

$1 - 2\,\text{pnorm}(-z_\alpha) = \alpha$

if $z_\alpha = -\text{qnorm}\left(\frac{1-\alpha}{2}\right)$

Formulas: Standard Normal

- $P(|Z| > a) = 2\,\text{pnorm}(-a)$
- $P(|Z| > z_\alpha) = \alpha$ if $z_\alpha = -\text{qnorm}(\alpha/2)$

- $P(|Z| < a) = 1 - 2\,\text{pnorm}(-a)$
- $P(|Z| < z_\alpha) = \alpha$ if $z_\alpha = -\text{qnorm}\left(\frac{1-\alpha}{2}\right)$

$X \sim \text{Normal}(\mu, \sigma)$

- $P(|X-\mu| > a) = 2\,\text{pnorm}(-a/\sigma)$
- $P(|X-\mu| > x_\alpha) = \alpha$ if $x_\alpha = -\sigma \cdot \text{qnorm}(\alpha/2)$

- $P(|X-\mu| < a) = 1 - 2\,\text{pnorm}(-a/\sigma)$
- $P(|X-\mu| < x_\alpha) = \alpha$ if $x_\alpha = -\sigma \cdot \text{qnorm}\left(\frac{1-\alpha}{2}\right)$

Ex: Compute $P(|Z| < 2)$

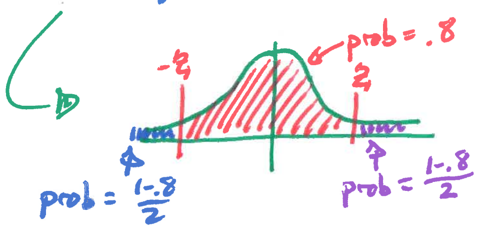
$1 - 2\,\text{pnorm}(-2) \approx 0.954$

Ex Compute $P(|Z| < 3)$

$1 - 2\,\text{pnorm}(-3) \approx 0.997$

Ex Find z so that $P(|Z| < z) = 0.8$

$-\text{qnorm}\left(\frac{1-.8}{2}\right) \approx 1.282$



Ex Compute $P(|X-2| > 3)$ if $X \sim \text{Normal}(2, 7)$

$2\,\text{pnorm}(-3/7)$

$= 2\,\text{pnorm}(-3, 0, 7)$

Ex Compute $P(|X-2| < 4)$ if $X \sim \text{Normal}(2, 7)$

$1 - 2\,\text{pnorm}(-4/7)$

$= 1 - 2\,\text{pnorm}(-4, 0, 7)$

Ex Find x so that $P(|X-2| > x) = 0.9$ if $X \sim \text{Normal}(2, 7)$

$-7 \cdot \text{qnorm}(.9/2)$

$= -\text{qnorm}(.9/2, 0, 7)$